# MATHEMATICS

# AND

# ART

BY FRAN AND HELEN

# PART ONE

# 1.1.INTRODUCTION

A brief introduction. Famous numbers in Mathematics (e.g. $\boldsymbol{\Phi} = \frac{1+\sqrt{5}}{2}$, e, $\Pi$ , i =$\sqrt{-1}$ , prime numbers, perfect numbers,amicable numbers,twin primes, Fibonacci sequence…)

# 1.2.THE GOLDEN NUMBER: $\Phi$

**The Partenon uses the golden number in its design of construction**

- Definition and basic properties.
- History of $\Phi$ in Art.
- $\Phi$ in Painting
- $\Phi$ in Architecture.
- $\Phi$ in Nature.
- $\Phi$ in Music.

# $\Phi$ IN KEOPS PYRAMID


- Some myths about $\Phi$, for example: does $\Phi$ appear “really” in the Pyramid of Keops or is it only “by chance”?

# 1.3.THE CORDOBESIAN RATIO

A number which has been discovered relatively few time ago and which is involved with Moslem Art in The Mosk of Córdoba (Spain).

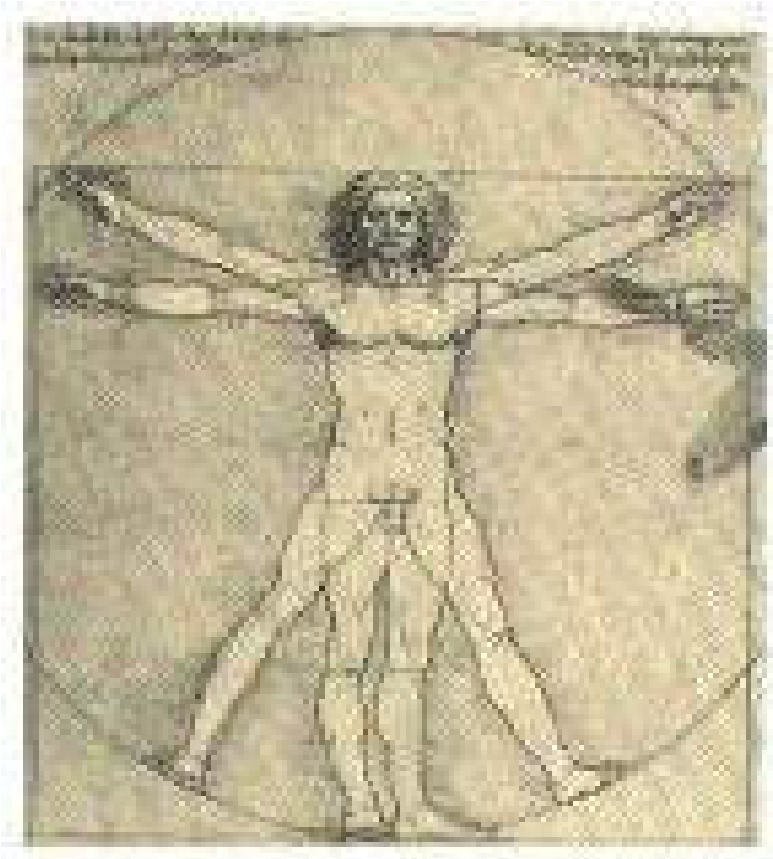
MOSK OF CÓRDOBA (FRONT)

MOSK OF CÓRDOBA (INSIDE)

# PART TWO

## *PRESENCE OF MATHEMATICS IN SEVERAL WORKS OF ART*

# 2.1. IN PAINTINGS



**“Vitruvio’s man” by Leonardo Da Vinci**.
The relation between the man’s high and the distance from the belly button to the fingers is the golden number.

- “Vitruvio’s man” by Leonardo da Vinci.
- “Saint Jeronimous in his cell” by A.Durero.
- “False Perspective” by Hogarth.
- “Frebcg Ambassadors” by Holbein.

# THE MELANCHOLY

**The Melancholy I by A.Durero**

It is said that Durero made this painting suffering a melancholy attack after his mother's death.

From 1495 Durero started to show a serious interest in Mathematics.

The femenine figure is holding an open compass in order to measure.

We can also see a magic square on the top right side. Each row, column, diagonal, the four central numbers and the numbers in the four corners sum up 34, which is a Fibonacci numberr.

The numbers in the middle of the last row make 1514, the date of the engraving.

We can also see a rhombusdohedron related to the Golden Number.

**“The Meninas” by Velázquez**

- “Las Meninas”by Velázquez.
- “Optics, Painting and Photography” by M.H.Pirenne.
- “Venus” by Botticelli (Gemalde Museum, Kulturforum, Berlin).
- “Fra Luca Paacioli and his pupil Guidobaldo”by Jacopo de Barbari.

# “THE MENINAS” BY PICASSO

**“The Meninas” by Picasso, 1957**

- **In the middle of the 50's, Picasso carried out some series of important pictures in the History of the Art.**
- **He painted a serie of fifty versions of The Meninas, in which everyone can realize the great admiration to Velázquez.**
- **The enormous figure representing Velázquez can be observed by everyone (from the floor to the ceiling).**
- **Picasso considered the painter more important than his work.**

# ANALYSING THE MENINAS BY PICASSO (1)

**One of the Meninas**

**The hand holding the tray**

**The tray with a vase**

# ANALYSING THE MENINAS BY PICASSO (2)

**The outline of the head**

**Right eye and nose**

**Left eye**

# ESCHER

**Maurits Cornelis Escher (1898-1972)**

was thoroughly intrigued by graphic arts. He explored the many and varied options which different graphic techniques made available, and he would choose his subject matter purely on the strength of the technical challenges it presented to his skills as a draughtsman.

His pictures reflected his fascination with the enigmatic laws governing the world around him.

**Escher's ring**

# 2.2.IN ARCHITECTURE

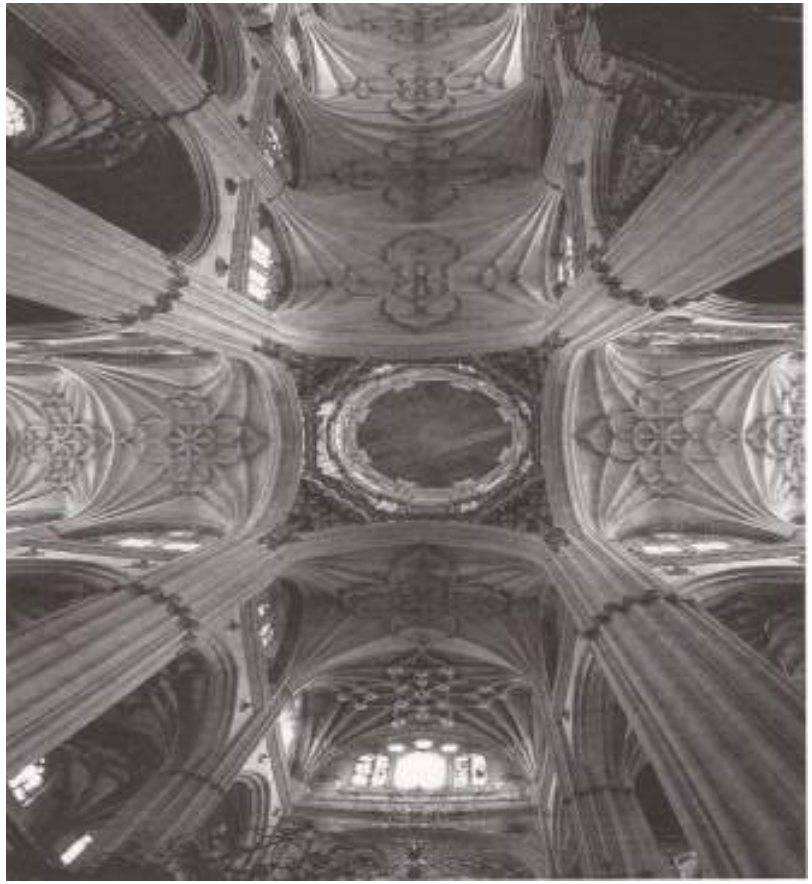
**Fran in “Neue National Galerie”(Berlin)**

- Fran and Helen looking for Geometry in Architecture.

**Helen in “Neue National Galerie”(Berlin)**

# GOTHIC AND ROMANESQUE GEOMETRY

**Gothic Geometry**



**Romanesque Geometry**

# 2.3.IN MUSIC

## B A C H

**The code in Johann Sebastian Bach's works**

# D S H

## The code in Dimitr Shostakovich's works

# PART THREE

## Art Room

# GYMKHANA

We can develop a kind of Gymkhana.

Some of these activities could be the following.

# 1.LOOKING FOR $\Phi$

The participant will have to find with a graduated rule the Golden ratio or the Cordobesian ratio in some works of art reproductions pictures.

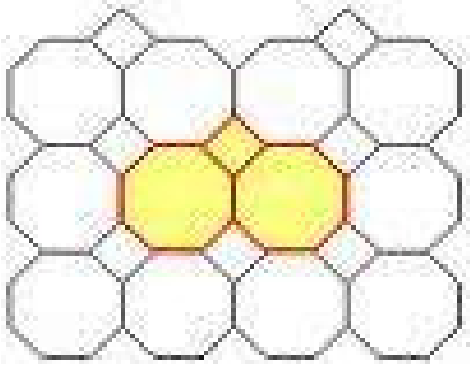
**Mudejar Geometry**

**Mosk of Córdoba(dome)**

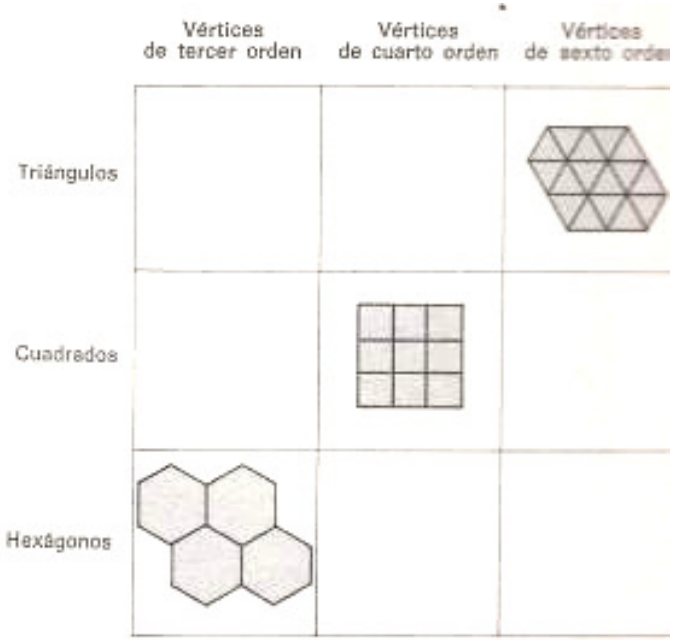
# 2.MOSAICS(1)

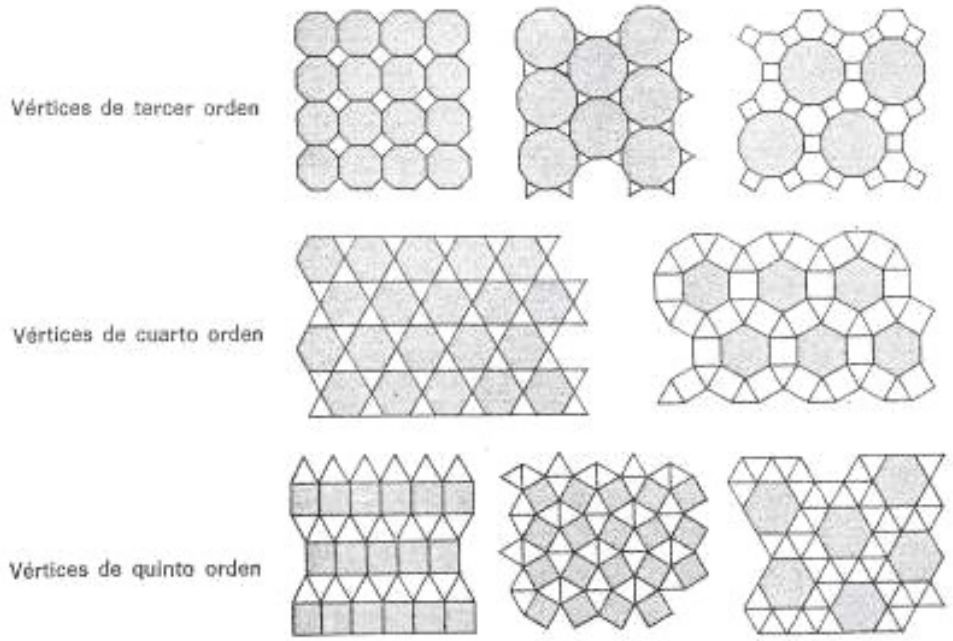
- We can only tile the plane with regular polygons in three different ways obtaining what is called regular mosaics.

- But if we combine regular polygons we can tile the plane, obtaining semiregular mosaics, in eight different ways.

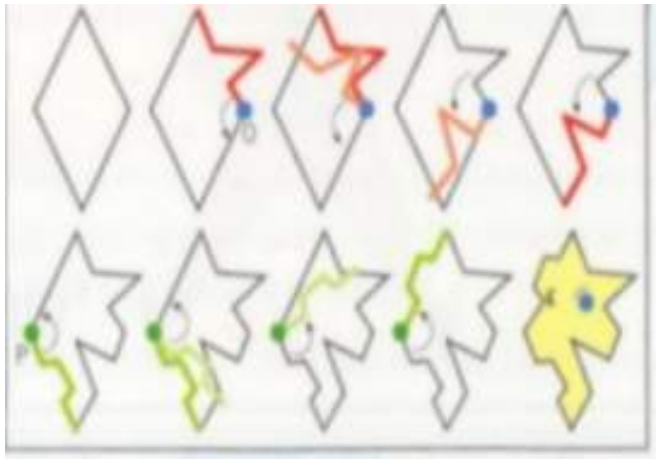


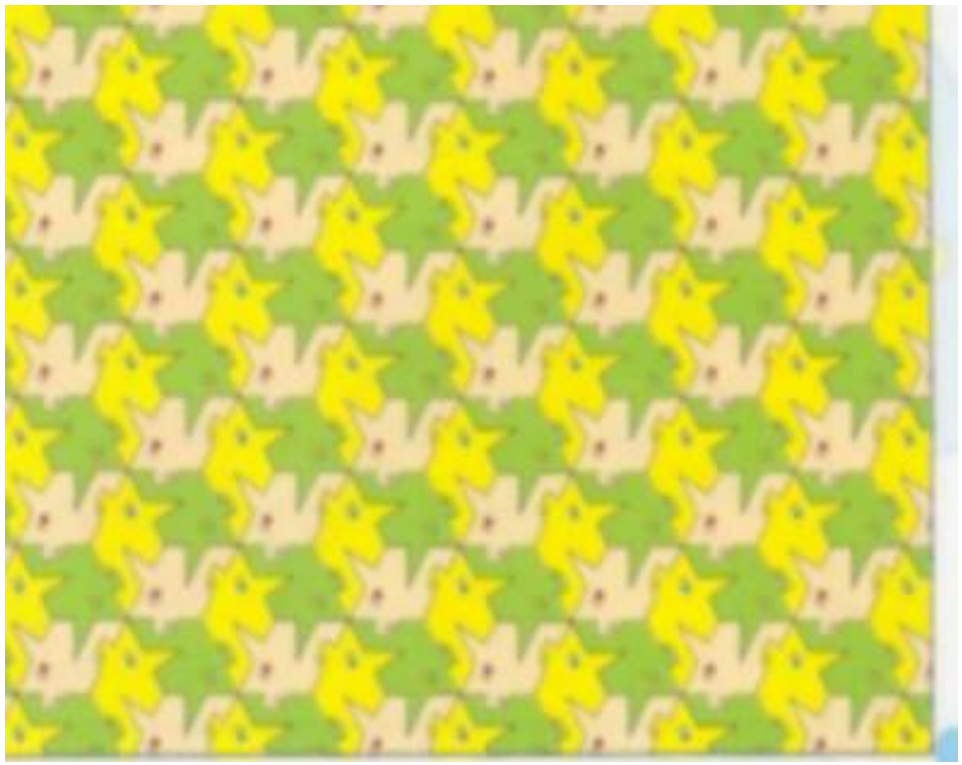
We give the participants sets of combined regular polygons and he/she has to find these semiregular mosaics.



Try to create your own mosaic taking a geometric figure and applying several deformations



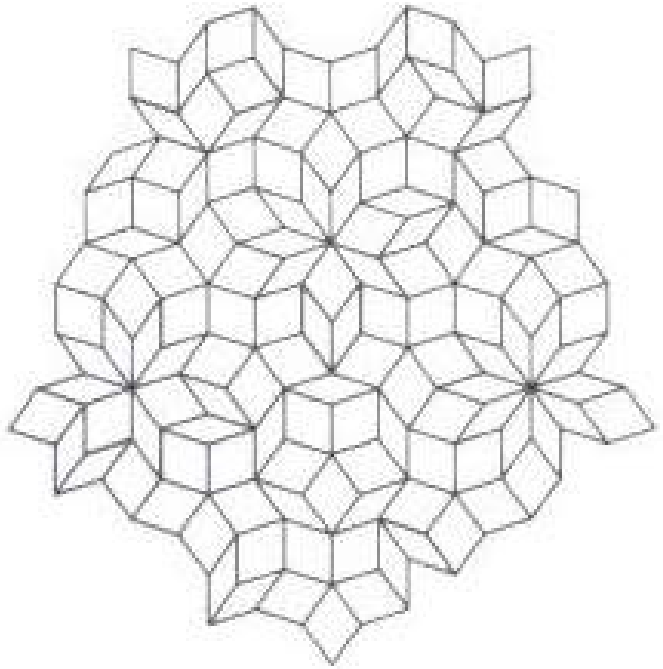


# 3.MOSAICS (2)

We can also do mosaics with non-regular polygons. For example, we can use rhombus.

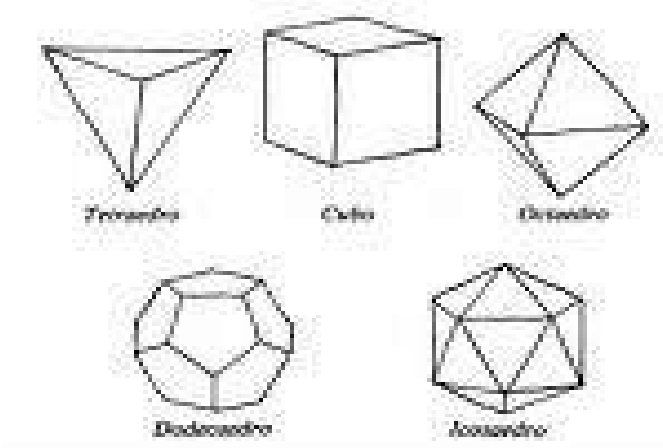
A rhombus is of course a two-dimensional figure, but with some particular rhombuses we can do figures which seem three-dimensional cubes projected onto the flat surface of the page.

Roger Penrose, a brilliant mathematician and physician proved that these pieces can be used to do non-periodic mosaics which tile the plane. In other words, we can fill a surface on the plane using these figures and we can do it without repeating the pattern.

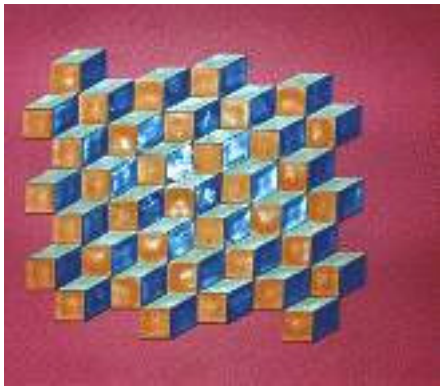


# 4.FILLING THE SPACE WITH POLYHEDRONS

There only exist five regular polyhedrons, the Platonic Solids: tetrahedron,cube, octahedron, dodecahedron and icosahedron.

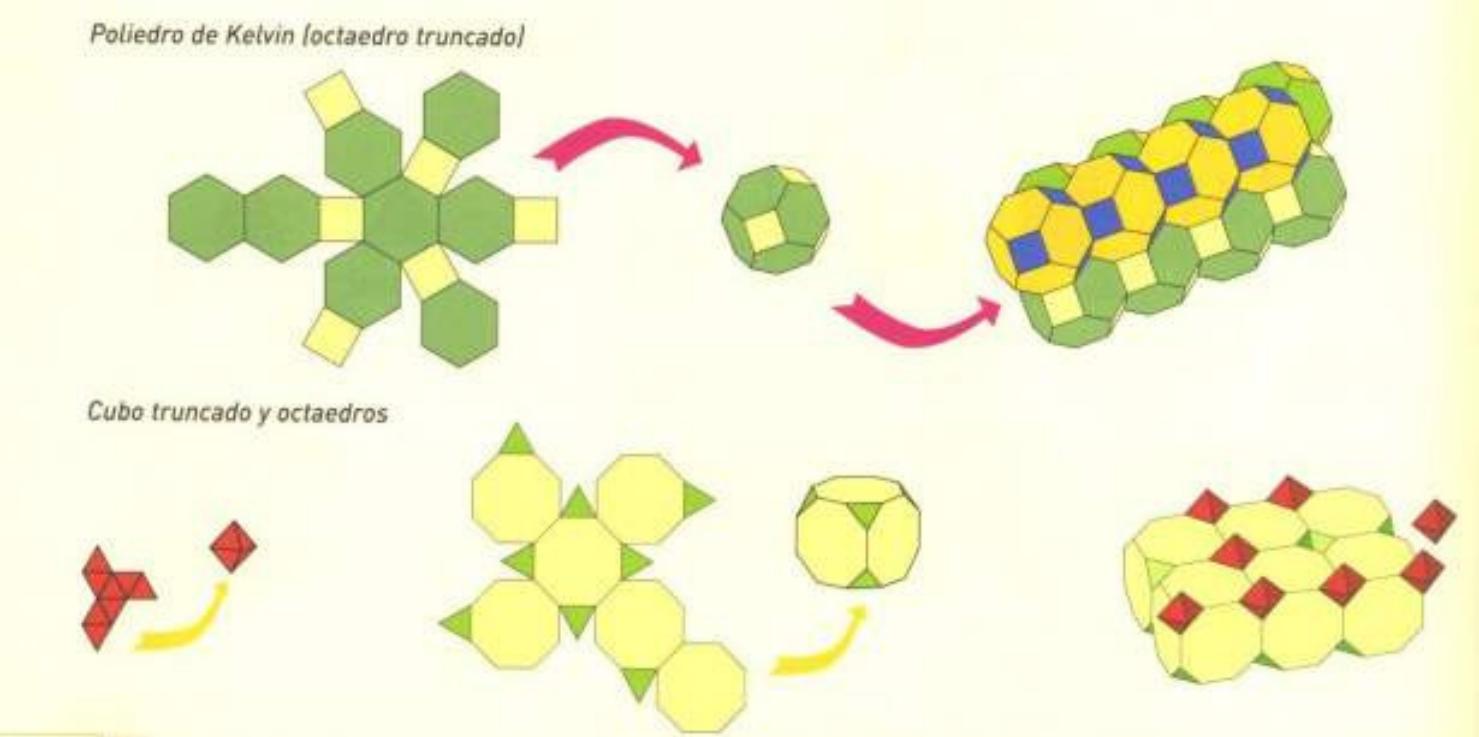


- Among these regular polyhedrons, only the cube can fil the spaces without holes.



- But if we combine regular polyhedrons and semiregular polyhedrons (Arquimedian Solids)we can fill the space without holes in 12 different ways.

These images show how to fill the space in some of these ways



# 5.A WITTY PROOF OF THE EQUALITY

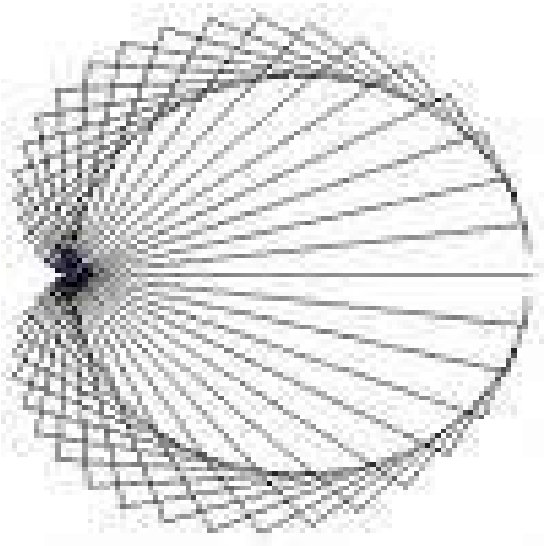
$$3^3 + 4^3 + 5^3 = 6^3$$

The participant is given several irregular pieces of wood built by cubes with edge 1cm. He/She has to find the way to make fix all these irregular pieces into one cube of edge 6cm (6x6x6).

Afterwards, he/she has to find the way to make fix all the pieces into three cubes- one of edge 3(3x3x3), one of edge 4 (4x4x4) and one of edge 5 (5x5x5).

# 6.DRAWING CURVES

By drawing systematically sets of paralell and perpendicular lines in a particular way, the participant will obtain some famous curves in Mathematics -the conics(parabola, hyperbola and elipse), the astroid, the epicycloid, hypocycloid, the cardiod,the Pascal's snail, the deltoid...

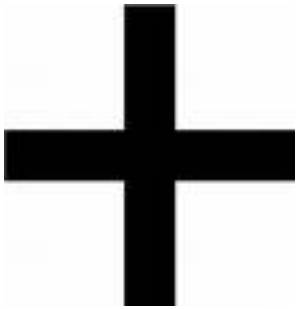


**The Cardiod**

# 7.BUILDING FIGURES WITH TANGRAM

The participant has to build some selected figures with a tangram, for instance, the Latin and Greek Crosses.

**Latin Cross**



**Greek Cross**

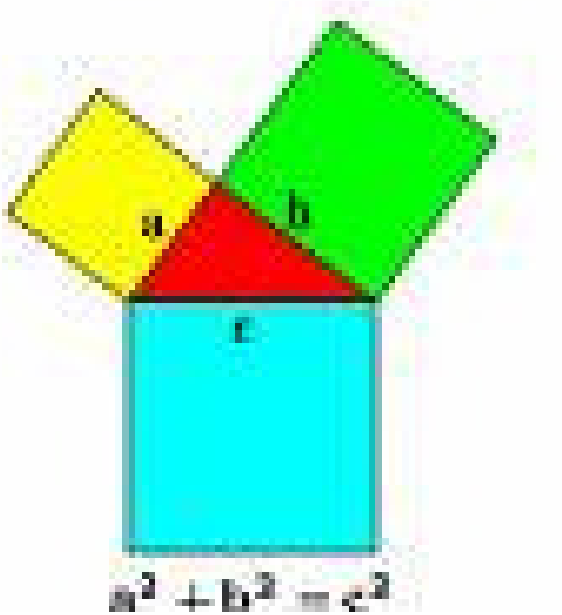
# 8.PROOFS WITHOUT WORDS

Watching some coloured figures and without writing anything the participant has to make sure he /she understand Pythagoras' Theorem or the two (torturing for children) identities:

$$(\alpha+\beta)\cdot(\alpha-\beta)=\alpha^2-\beta^2$$

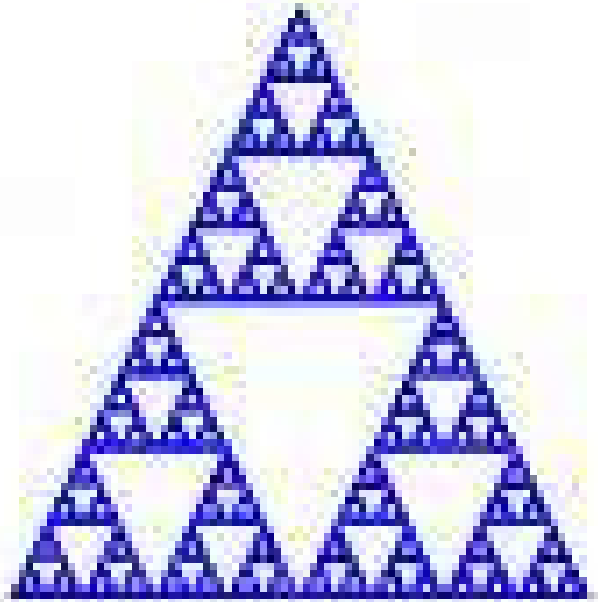
$$(a+b)^2=a^2+2ab+b^2$$

**Pythagoras' Theorem**

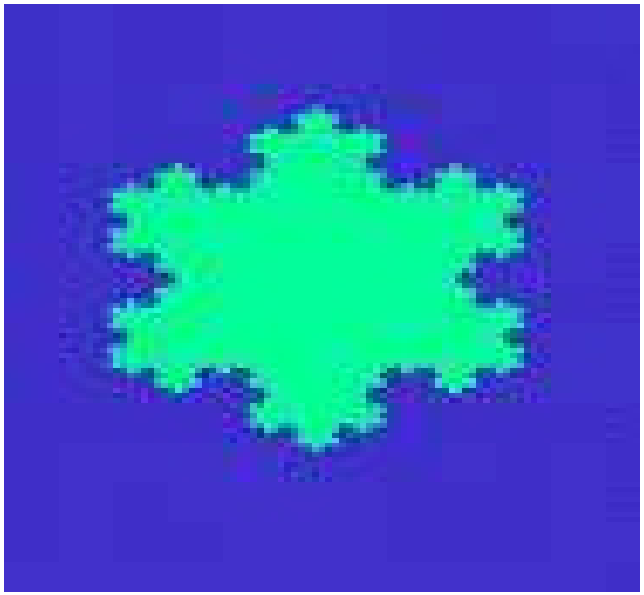


# 9.FRACTALS

The participant can create his/her own fractals with a rule and crayons, or with the personal computer.



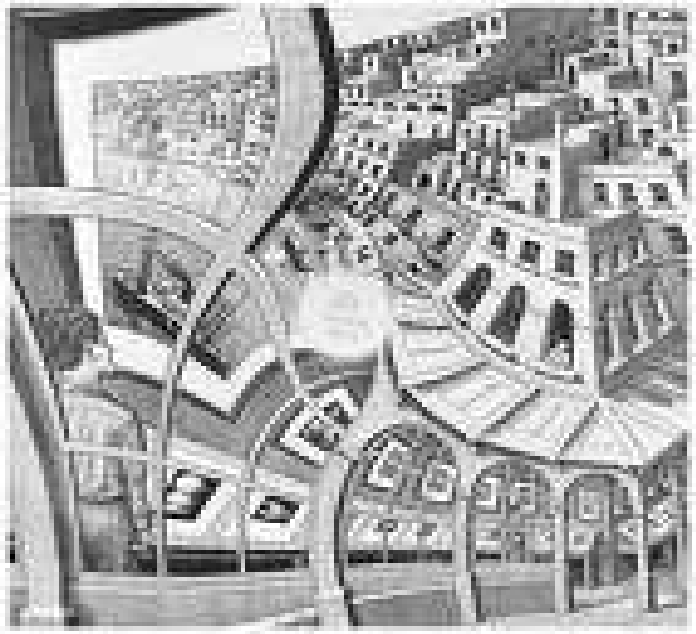
**Serpinsky triangle**

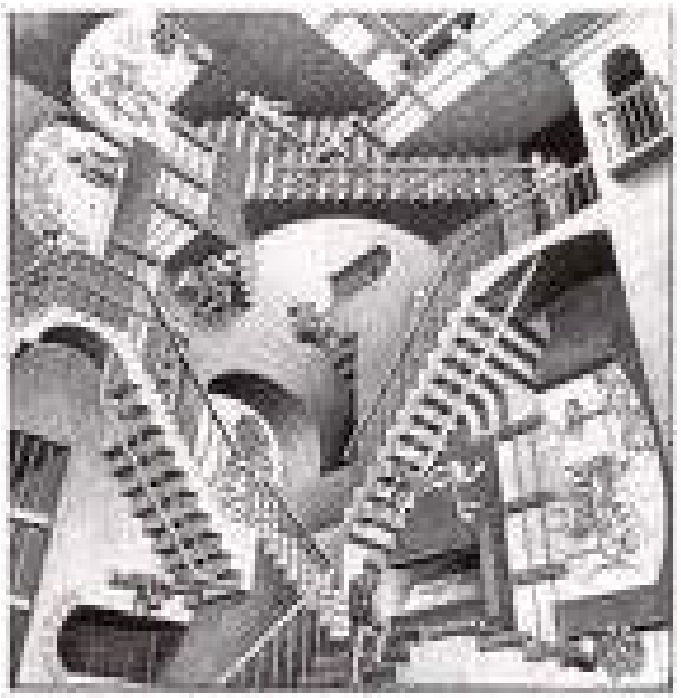


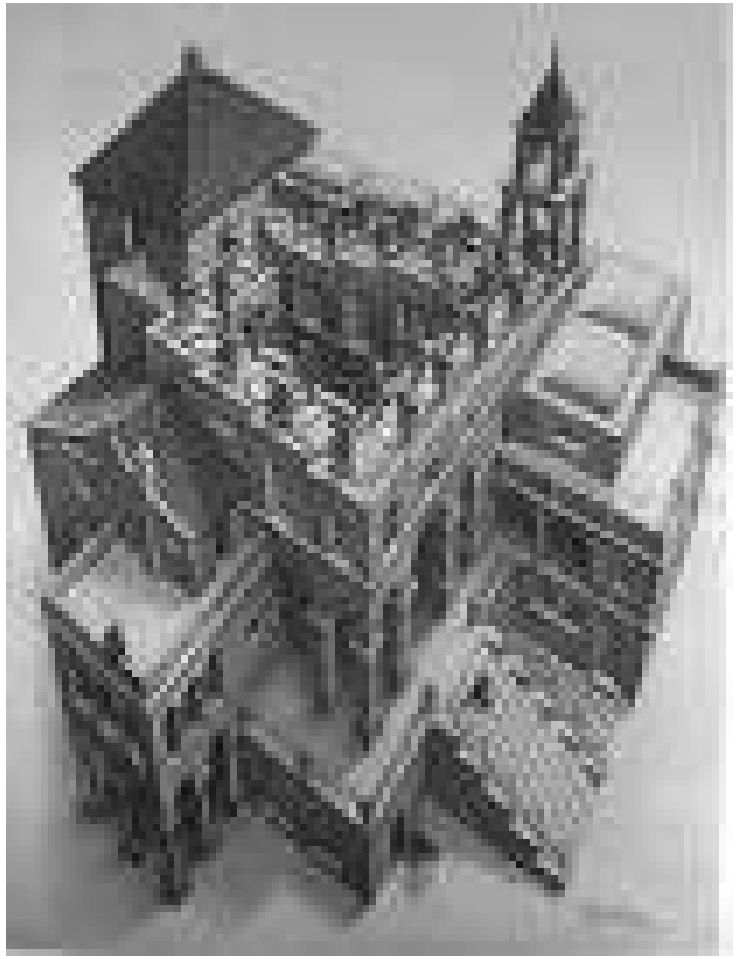
**snowflake**

# 10.IS IT POSSIBLE?

Analysing Escher's works







# THE END